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A HYBRID MODELING APPROACH: DYNAMIC SIMULATION OF TWO-PHASE FLUID FLOW IN COMPACTING GAS CONDENSATE RESERVOIRS USING POTENTIAL FLOW THEORY

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ABSTRACT

Simulating the flow processes of complex hydrocarbon mixtures, such as gas condensate mixtures and volatile oils, in deep reservoirs is one of the most challenging problems in reservoir hydrodynamics. The challenge lies in meeting three requirements simultaneously when creating a simulator: high accuracy, low computational cost, and high reliability. These metrics determine the performance of the simulation. Meeting all these requirements at the same time necessitates a hybrid approach to solving the problem and optimizing the computational algorithm in terms of CPU time. A new technique has been developed for hybrid modeling of the development of deposits of complex hydrocarbon mixtures. This technique integrates the theory of potential flow, the Binary Flow Model (which accounts for rock compaction, PVT properties of reservoir fluids, phase transformation, and mass transfer between phases), and the material balance equations of the hydrocarbon system using time discretization. In this case, to linearize nonlinear differential equations for the flow of a gas-condensate mixture, the method of averaging reservoir pressure along the radial coordinate was used. By introducing the Khrestianovich function, an algorithm was obtained to determine the rate of inflow of the gas-condensate mixture to the well. Using material balance equations for gas and condensate, it was possible to obtain differential equations that describe changes in reservoir pressure and condensate saturation over time. Based on this technique, a simulator for a gas condensate reservoir was created, enabling computer studies to be conducted. The results demonstrated the proposed technique's good accuracy compared with the results of the semi-analytical solution.

KEYWORDS:
Dynamic modeling;
Simulation;
Monitoring;
Streamline;
Hybrid modeling;
Binary model.

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1. Introduction

In order to calculate flow processes in big deep reservoir with complex hydrocarbon mixtures, fast solutions for reservoir dynamics are needed, since not only the PVT properties and physical phase changing processes are complex but also computational domains are large. The classical Integral Finite Difference methods could require millions of elements making the numerical method computationally expensive. One approach to overcome this problem is to use the theory of potential flow. The fundamentals of potential flow theory can be traced back to the 19th century [2]. Modeling fluid flow and transport on the bases of the potential flow theory dates back to the seminal work of Muskat [8]. Since then, several authors have applied and extended the underlying concepts for applications to petroleum reservoir modeling [9-15].

Streamline method has a number of advantages over Integral-Finite-Difference Simulation: It requires fewer data and fewer computational resources, hence it is easy and fast to implement; it is faster and works in real-time [2]. Using streamline method as a basis, and to predict the parameters on the external reservoir boundary, it is possible to apply the material balance equations and the solution of the one-dimensional flow problem to determine the flow rates in any cell of the reservoir. In this case, the simulation performance will depend from the representation of the complex hydrocarbon mixture. For example, a gas-condensate mixture (like volatile oil) is a complex hydrocarbon fluid system. Its gas contains heavy hydrocarbon components which condensates with decreasing reservoir pressure below the dew point, leading to the formation of a liquid phase (when, in volatile oil, lighter components evaporate and a gas phase is created) in the reservoir. This leads to a two-phase hydrocarbon system in the reservoir. The mass transfer of components takes place between these phases. In this sense, there are basically two approaches to the hydrodynamic modeling of the flow process of complex hydrocarbon systems, such as

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gas condensate mixture, and volatile oil. According to the first approach, the movement of the hydrocarbon mixture is considered by analogy with the movement of dissolved gas in live oil (dissolved condensate in gas, similar to the Black-oil) model. According to the second approach, the hydrocarbon mixture is represented as a two-phase multi-component hydrocarbon system. While the live oil model is easier and more accessible to users, its results are not as accurate as in the compositional model. However, the compositional model requires many parameters, which makes mathematical modeling more difficult. Therefore, it is not always applicable in practice. The multi-component model has several varieties, one of which is the Binary model. The Binary model represents the complex hydrocarbon systems consisting of two mutually soluble pseudo-components and two phases, between which the components are allocated.

The following section proposes a technique based on the Binary Model and the Potential Flow Theory for simulating the development of a reservoir containing a complex hydrocarbon mixture, using gas condensate as an example. The results of the computer simulation developed with this technique are also presented.

2. Mathematical model of the gas-condensate mixture flow through compacting reservoir rocks

Within the framework of the Binary model, the movement of the gas-condensate mixture (not taking bound water into account) can be represented by the following system of differential equations [7]:

$$\frac{1}{r}\frac{\partial}{\partial r}\left[r\varphi_g(p,s)\frac{\partial p}{\partial r}\right]=-\frac{\partial}{\partial t}f_g(p,s) \quad (1)$$

$$\frac{1}{r}\frac{\partial}{\partial r}\left[r\varphi(p,s)\frac{\partial p}{\partial r}\right]=-\frac{\partial}{\partial t}f(p,s) \quad (2)$$

where

$$\begin{aligned}\varphi_g(p,s)&=\left[\frac{k_{rg}(s)\,p\beta\left[1-c(p)\,\overline{\gamma}(p)\right]}{\mu_g(p)z(p)p_{at}}+\frac{k_{ro}(s)R_s(p)}{\mu_o(p)B_o(p)}\right]k(p),\\ \varphi(p,s)&=\left[\frac{k_{ro}(s)}{\mu_o(p)B_o(p)}+\frac{k_{rg}(s)p\beta c(p)}{\mu_g(p)z(p)p_{at}}\right]k(p),\\ f(p,s)&=\left[\frac{s}{B_o(p)}+(1-s)\frac{p\,\beta c(p)}{z(p)p_{at}}\right]\phi(p),\\ f_g(p,s)&=\left[\frac{(1-s)p\,\beta\left[1-c(p)\,\overline{\gamma}(p)\right]}{z(p)p_{at}}+s\frac{R_s(p)}{B_o(p)}\right]\phi(p)\end{aligned} \quad (3)$$

$k_{ro}(s)$ and $k_{rg}(s)$ are relative phase permeability for liquid and gas phases respectively; s is the pore saturation with liquid phase (i.e., condensate); z, β are gas compressibility factor and temperature correction for gas phase respectively; c is the content of potentially liquid hydrocarbons in the gas phase; μ_o, μ_g are viscosities of liquid and gas phases respectively; B_o is the formation volume factor of the liquid phase; R_s is the amount of dissolved gas in the liquid phase; $\overline{\gamma}=\frac{\gamma_o(p)}{\gamma_g(p)}$ is the ratio of specific gravities of liquid and gas phases at reservoir pressure (p); p_{at} is atmospheric pressure; ϕ is the reservoir porosity depending on reservoir pressure (p); k is the reservoir permeability depending on reservoir pressure (p); r is a radial coordinate; t is a time.

Equations (1) and (2) describe the radial motion of the gas-condensate mixture. (1) is the equation of motion of gas and the gas dissolved in the condensate. In the beginning, there is no liquid phase (condensate). With a drop-in pressure, at a lower condensation pressure (i.e. dew point), a retrograde condensation phenomenon occurs and a liquid phase is formed in the reservoir.

Equation (2) describes the radial motion of the liquid phase (condensate) and potential condensate, i.e. heavier hydrocarbons (C_{5+} components) dissolved in gas.

Since the main product of the gas condensate reservoir is gas, it is necessary to solve equation (1) to determine the well production rate. However, equation (1) is non-linear. For an analytical solution, it is necessary to linearize it. For this purpose, we use an averaging method, according to which the average pressure along the formation (in the [r_w- R_e] range) is taken for the pressure on the right side of the equation. This method is quite accurate for changing the reservoir pressure in a wide range [1].

This leads to the fact that the right-hand side of the equation depends only on time by $F(t)$ function, which is further determined by the additional boundary condition:

$$\frac{1}{r}\frac{\partial}{\partial r}\left[r\varphi_g(p,s)\frac{\partial p}{\partial r}\right]=-F(t)$$

By introducing a pseudo-pressure function $H=\int\varphi_g(p,s)dp+const$ this equation can be reduced to the following formula:

$$\frac{1}{r}\frac{\partial}{\partial r}\left\{r\frac{\partial H}{\partial r}\right\}=-F(t) \quad (4)$$

Solving (4) with boundary conditions $r=R_e$, $H=H_e(t)$ $r=r_w$; $H=H_w(t)$ and $\left.\frac{\partial H}{\partial r}\right|_{r=R_e}=0$, the following expression for the distribution of the H function along the r coordinate (common solution) was obtained:

$$H=\frac{H_w-H_e}{R_e^2\left(2\ln\frac{R_e}{r_w}-1\right)+r_w^2}\cdot\left[R_e^2\left(2\ln\frac{R_e}{r}-1\right)+r^2\right]+H_e \quad (4^*)$$

Therefore, from (4*) we obtain the gradient of the

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function H in the following form:

$$\frac{\partial H}{\partial r}=\frac{H_e-H_w}{\frac{R_e^2}{2}\left(2\ln\frac{R_e}{r_w}-1\right)+\frac{r_w^2}{2}}\left(\frac{R_e^2-r^2}{r}\right) \qquad (4^{**})$$

According to Darcy's law, the well flow rate is determined by the following formula:

$$q=S\nu=2\pi r_w h\cdot\left.\frac{\partial H}{\partial r}\right|_{r=r_w}$$

here S, h, ν – the wellbore cross-sectional area, formation thickness, flow velocity of gas condensate mixture into the well from the reservoir respectively. Thus, considering (4**), we obtain expressions for determining the well production rate in the following form [17]:

$$q=\frac{2\pi h\left(H_e-H_w\right)\left(1-\frac{r_w^2}{R_e^2}\right)}{\ln\frac{R_e}{r_w}-\frac{1}{2}\left(1-\frac{r_w^2}{R_e^2}\right)} \text{ or since } \frac{r_w^2}{R_e^2}\ll 1, \text{ then}$$

$$q=\frac{2\pi h(H_e-H_w)}{\ln\frac{R_e}{r_w}-\frac{1}{2}} \qquad (5)$$

where R_e, r_w are the radiuses of the reservoir boundary (or well drainage zone) and the well respectively.

Expression (5) allows solving well-test data and well production data related problems. However, a transition from pseudo-pressure H to the actual pressure p is required to apply the above expression. For this purpose, the φ function was investigated, and it was established that it could be approximated with a high degree of accuracy by a polynomial of the second degree:

$$\varphi(p,s)=ap^2+bp+c \qquad (6)$$

This idea is confirmed by data presented in figure 1. It depicts changes of $\varphi(p, s)$ versus reservoir pressure. The latter is calculated from equations (3) and (6) correspondingly. As can be seen from figure 1, the curves almost coincide. Then, taking into account approximation (6), we could integrate functions $H=\int\varphi_g(p,s)dp+const$ within pressure limits $[p_w, p_e]$, so we could obtain the expressions for computing the pseudo depression H_k-H_s in the following form:

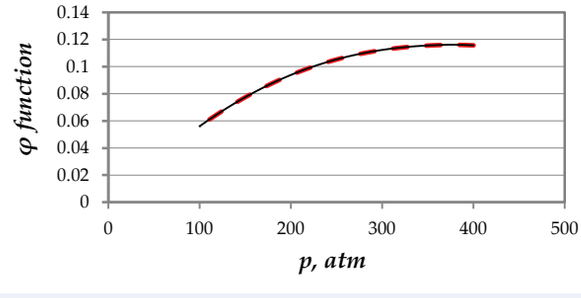

Fig. 1. Comparison of curves of change in φ versus pressure: red dotted line- calculated numerically by (3) and solid line- by the approximating polynomial (6)

$$H_e-H_w=\frac{a}{3}(p_e^3-p_w^3)+\frac{b}{2}(p_e^2-p_w^2)+c(p_e-p_w) \qquad (7)$$

Here coefficients a, b, c are determined by values of φ function at the well bottomhole (i.e. at bottomhole pressure) and on the reservoir boundary (i.e. at reservoir pressure) in the following way:

$$a=\frac{(\tilde{\varphi}-\varphi_w)(p_e-p_w)-(\varphi_e-\varphi_w)(\tilde{p}-p_w)}{(p_e-p_w)(\tilde{p}-p_w)(\tilde{p}-p_e)},$$
$$b=\frac{(\varphi_e-\varphi_s)-a(p_e^2-p_w^2)}{p_e-p_w},\quad c=\varphi_w-ap_w^2-bp_w \qquad (8)$$

where φ_e, φ_w and $\tilde{\varphi}$ are the values of the integrand at reservoir, bottomhole and average reservoir pressures respectively; $\tilde{\varphi}=\varphi(\tilde{p},\tilde{s})$, $\tilde{p}=\frac{p_e+p_w}{2}$.

Considering (7), the expression for determining the inflow to the well (5) takes the following form [5]:

$$q=\frac{2\pi h\left[\frac{a}{3}(p_e^3-p_w^3)+\frac{b}{2}(p_e^2-p_w^2)+c(p_e-p_w)\right]}{\ln\frac{R_e}{r_w}-\frac{1}{2}} \qquad (9)$$

Expression (9) provides to determine the well production rate at a given reservoir pressure. To calculate the reservoir pressure and the production rate at the next time step, the material balance equations of the hydrocarbon system will be used. The next section is devoted to this problem.

3. Material balance equations

Below we get an algorithm to calculate the values of reservoir pressure, condensate saturation, and the total reservoir pore volume. With this in view, we will use the material-balance equations of the liquid and gas phases of the hydrocarbon system [6]:

$$q_o=-\frac{d}{dt}\left[\frac{s}{B_0(p)}+(1-s)\frac{p\beta c(p)}{z(p)p_{at}}\right]\Omega \qquad (10)$$

$$q_g=-\frac{d}{dt}\left[\frac{(1-s)p\beta}{z(p)p_{at}}[1-c(p)\,\bar{\gamma}]+s\frac{R_s(p)}{B_o(p)}\right]\Omega \qquad (11)$$

Based on equations system (10) and (11), we obtain a system of differential equations describing temporal variation in reservoir pressure and oil saturation state [1, 3, 16]:

$$\frac{dp}{dt}=-\frac{\frac{q_o}{\Omega_0\bar{\Omega}}(\alpha_4+G\alpha_2)-(\alpha_2\alpha_3+\alpha_1\alpha_4)\frac{1}{\bar{\Omega}}\frac{d\bar{\Omega}}{dt}}{(\alpha_5+\alpha_6)\alpha_4+(\alpha_7+\alpha_8)\alpha_2} \qquad (12)$$

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$$\frac{ds}{dt} = -\frac{\frac{q_o G}{\Omega_0 \bar{\Omega}} + (\alpha_7 + \alpha_8)\frac{dp}{dt} + \alpha_3 \frac{1}{\bar{\Omega}}\frac{d\bar{\Omega}}{dt}}{\alpha_4} \quad (13)$$

where oil production rate (q_o) is calculated by (11);

$$\Omega(p,t) = \pi(R_e^2 - r_w^2)h\phi(p); \quad \bar{\Omega} = \frac{\Omega}{\Omega_0}$$

$$\Omega_0 = \pi(R_e^2 - r_w^2)h\phi(p_0);$$

$$\alpha_1 = (1-s)\frac{p\beta c(p)}{z(p)p_{at}} - s\frac{1}{B_o(p)}, \quad \alpha_2 = \frac{p\beta c(p)}{z(p)p_{at}} - \frac{1}{B_o(p)},$$

$$\alpha_3 = s\frac{R_s(p)}{B_o(p)} - (1-s)\frac{p\beta}{z(p)p_{at}}[1 - c(p)\,\bar{\gamma}(p)]$$

$$\alpha_4 = \frac{R_s(p)}{B_o(p)} - \frac{p\beta}{z(p)p_{at}}[1 - c(p)\,\bar{\gamma}(p)],$$

$$\alpha_5 = (1-s)\left\{\frac{p\beta c(p)}{z(p)p_{at}}\right\}', \quad \alpha_6 = s\left[\frac{1}{B_o(p)}\right]',$$

$$\alpha_7 = s\left[\frac{R_s(p)}{B_o(p)}\right]', \quad \alpha_8 = (1-s)\left[\frac{p\beta}{z(p)p_{at}}[1 - c(p)\,\bar{\gamma}(p)]\right]'$$

Here «′» means the derivative with respect to p; $\bar{\mu} = \frac{\mu_o(p)}{\mu_g(p)}$ – viscosity ratio of liquid and gas phases; $\psi = \frac{k_{rg}(s)}{k_{ro}(s)}$ – the ratio of the relative phase permeability of the gas and liquid phases; G – the gas oil ratio, defined by the following expression:

$$G(p) = \frac{\frac{\bar{\mu}(p)B_o(p)p\beta}{z(p)p_{at}}[1 - c(p)\bar{\gamma}(p)] + \frac{R_s(p)}{\psi(s)}}{\frac{1}{\psi(s)} + \frac{\bar{\mu}(p)B_o(p)p\beta c(p)}{z(p)p_{at}}} \quad (14)$$

As you know, a decrease in pore pressure results in deformation of the reservoir under rock pressure, which leads to a change in the porosity and permeability of the reservoir. The formation deformation can be elastic, elastic-viscous or viscous depending on the rheological properties of the rocks. This paper considers an elastic formation. The changes in porosity and permeability under elastic deformation obey the exponential law and are determined by the following expressions [5]:

$$\phi = \phi_0 \exp\left[a_m\left(p - p_0\right)\right] \text{ and } k = k_0 \exp\left[\beta_k\left(p - p_0\right)\right] \quad (15)$$

Here β_k and a_m, are coefficients of permeability and porosity change, respectively; ϕ_0, k_0, ϕ, k – are initial and current values of porosity and permeability, respectively; p_0 – is initial reservoir pressure.

Porosity (15) expression allows us to get:

$$\bar{\Omega} = \frac{\Omega}{\Omega_0} = \exp\left[a_m(p - p_0)\right] \text{ and}$$
$$\frac{d\bar{\Omega}}{dt} = a_m \exp\left[a_m(p - p_0)\right]\frac{dp}{dt} \quad (16)$$

From (1), (2), and (14), we can obtain an expression for saturation on the reservoir external boundary:

$$\frac{ds}{dp} = \frac{(1-s)\left(\phi(p)\frac{p\beta}{z(p)p_{at}}\left[1 - c(p)\bar{\gamma}\right]\right)' + s\left(\phi(p)\frac{R_s(p)}{B_o(p)}\right)' - }{\phi(p)\left(\frac{p\beta}{z(p)p_{at}}\left[1 - c(p)\bar{\gamma}\right] - \frac{R_s(p)}{B_o(p)}\right) + } \rightarrow$$
$$\rightarrow \frac{-\left\{s\left(\frac{\phi(p)}{B_o(p)}\right)' + (1-s)\left(\phi(p)\frac{p\beta c}{z(p)p_{at}}\right)'\right\}G(p)}{+\left(\frac{1}{B_o(p)} - \frac{p\beta c(p)}{z(p)p_{at}}\right)\phi(p)G} \quad (17)$$

where $G(p)$ is determined by (14).

To determine the recovery factor, we need to calculate the initial reserves of hydrocarbons. For a gas condensate deposit, they are determined by the following formulas:

$$V_{g0} = \pi R_e^2 h m_0 \frac{p_0\beta\left[1 + c(p_0)\,\bar{\gamma}(p_0)\right]}{z(p_0)p_{am}} \quad \text{and}$$
$$V_{c0} = \frac{V_{g0}}{G(p_0)} \quad (18)$$

where GOR: $G(p_0) = \frac{1 - c(p_0)\bar{\gamma}(p_0)}{c(p_0)}$

Gas recovery and condensate recovery factors are determined by the following formulas:

$$GRF = \frac{Q_g}{V_{g0}} \text{ and } CRF = \frac{Q_c}{V_{c0}}$$
$$\text{where } Q_g = \sum_{i=0}^{n} q_{gi}\Delta t \text{ and } Q_c = \sum_{i=0}^{n} q_{ci}\Delta t \quad (19)$$

The system of ordinary differential equations (12), (13) and (16) allows us to determine the change in the reservoir pressure, saturation and total pore volume (p, s, Ω, respectively) for the next time step. (3) describes the dependence of saturation on reservoir pressure.

4. Potential Flow Theory

Introduces in the 1950s by [4], the Potential Flow Theory is an efficient simulation method attracting renewed interest. The presence of modern mathematical and computational capabilities opens up new application possibilities. Having the principles of Binary modeling in combination with streamline technology allows to achieve highly efficient simulation of the development process of hydrocarbon reservoirs with complex PVT properties, such as gas condensate and volatile oils. Below we consider the application of the potential flow theory to the case of flow of a gas condensate mixture.

Assume that there are n wells with bottom hole potential Φ_{ci} (where i=1,2,...,n) in a horizontal reservoir with thickness h. For n wells, the potential at any

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point of reservoir (A) is determined by the formula [4]:

$$\Phi = \Phi_1 + \Phi_2 + \cdots + \Phi_n = \frac{1}{2\pi h}\sum q_i \ln r_i + c \qquad (20)$$

By placing point A sequentially at the bottom of each well, we obtain the expression for the bottomhole potential for each of them:

$$\begin{cases} \Phi_{c_1} = \frac{1}{2\pi h}\left(q_1 \ln r_{c1} + q_2 \ln r_{12} + q_3 \ln r_{13} + \cdots + q_n \ln r_{1n}\right) + c \\ \Phi_{c2} = \frac{1}{2\pi h}\left(q_1 \ln r_{21} + q_2 \ln r_{c1} + q_3 \ln r_{23} + \cdots + q_n \ln r_{2n}\right) + c \\ \cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots \\ \Phi_{cn} = \frac{1}{2\pi h}\left(q_1 \ln r_{n1} + q_2 \ln r_{n2} + q_3 \ln r_{n3} + \cdots + q_n \ln r_{cn}\right) + c \end{cases} \qquad (21)$$

System (21) consists of n equations and contains $n+1$ unknowns (n well flow rate and integration constant c). An additional equation is obtained by placing point A on the reservoir edge:

$$\Phi_k \approx \frac{1}{2\pi h}\left(q_1 \ln r_k + q_2 \ln r_k + \cdots + q_n \ln r_k\right) + c \qquad (22)$$

By subtracting each of the equations (21) from (22) term by term, we eliminate the constant c and obtain a system of n equations. By solving n equations, it is possible to determine the flow rates of wells q_1, q_2, ..., q_n if the bottomhole and contour potentials Φ_{c_1}, Φ_{c_2}, ..., Φ_{c_n} and Φ_k are given, respectively.

$$\text{Since } \Phi = H(p), \text{ then } \Phi_k = H(p_k),\ \Phi_c = H(p_c) \qquad (23)$$

After subtractions and substitution (23), we obtain a system of n equations in the form:

$$\begin{cases} H(p_k) - H(p_{c1} + \Delta p_{01}) = \frac{1}{2\pi h}\left(q_1 \ln \frac{r_k}{r_{c1}} + q_2 \ln \frac{r_k}{r_{12}} + \cdots + q_n \ln \frac{r_k}{r_{1n}}\right) \\ H(p_k) - H(p_{c2} + \Delta p_{02}) = \frac{1}{2\pi h}\left(q_1 \ln \frac{r_k}{r_{21}} + q_2 \ln \frac{r_k}{r_{c2}} + \cdots + q_n \ln \frac{r_k}{r_{2n}}\right) \\ \cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots \\ H(p_k) - H(p_{cn} + \Delta p_{0n}) = \frac{1}{2\pi h}\left(q_1 \ln \frac{r_k}{r_{n1}} + q_2 \ln \frac{r_k}{r_{n2}} + \cdots + q_n \ln \frac{r_k}{r_{cn}}\right) \end{cases} \qquad (24)$$

or

$$\begin{cases} q_1 \ln \frac{r_k}{r_{c1}} + q_2 \ln \frac{r_k}{r_{12}} + \cdots + q_n \ln \frac{r_k}{r_{1n}} = 2\pi h\left(H(p_k) - H(p_{c1} + \Delta p_{01})\right) \\ q_1 \ln \frac{r_k}{r_{21}} + q_2 \ln \frac{r_k}{r_{c2}} + \cdots + q_n \ln \frac{r_k}{r_{2n}} = 2\pi h\left(H(p_k) - H(p_{c2} + \Delta p_{02})\right) \\ \cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots \\ q_1 \ln \frac{r_k}{r_{n1}} + q_2 \ln \frac{r_k}{r_{n2}} + \cdots + q_n \ln \frac{r_k}{r_{cn}} = 2\pi h\left(H(p_k) - H(p_{cn} + \Delta p_{0n})\right) \end{cases} \qquad (25)$$

Substituting the below expressions

$$c_1 = 2\pi h\left(H(p_k) - H(p_{c1} + \Delta p_{01})\right)$$
$$c_2 = 2\pi h\left(H(p_k) - H(p_{c2} + \Delta p_{02})\right)$$
$$\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots$$
$$c_n = 2\pi h\left(H(p_k) - H(p_{cn} + \Delta p_{0n})\right)$$

$$a_{11} = \ln \frac{r_k}{r_{c1}},\ a_{12} = \ln \frac{r_k}{r_{12}},\ \cdots,\ a_{1n} = \ln \frac{r_k}{r_{1n}}$$
$$a_{21} = \ln \frac{r_k}{r_{21}},\ a_{22} = \ln \frac{r_k}{r_{c2}},\ \cdots,\ a_{2n} = \ln \frac{r_k}{r_{2n}} \qquad (26)$$
$$\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots$$
$$a_{n1} = \ln \frac{r_k}{r_{n1}},\ a_{n2} = \ln \frac{r_k}{r_{n2}},\ \cdots,\ a_{nn} = \ln \frac{r_k}{r_c}$$

here

$$r_{c1} = r_{c2} = \cdots = r_{cn} = r_c\sqrt{e}, r_{c1} = r_{c2} = \cdots = r_{cn} = r_c\sqrt{e},$$
$$r_{n1} = r_{n2} = \cdots = r_{nn} = r_n\sqrt{e}, r_{1n} = r_{2n} = \cdots = r_{nn} = r_n\sqrt{e}$$

in (25) we obtain a system of n equations for determining the flow rates q_1, q_2, ..., q_n in the form:

$$\begin{cases} q_1 a_{11} + q_2 a_{12} + \cdots + q_n a_{1n} = c_1 \\ q_1 a_{21} + q_2 a_{22} + \cdots + q_n a_{2n} = c_2 \\ \cdots\cdots\cdots\cdots\cdots\cdots \\ q_1 a_{n1} + q_2 a_{n2} + \cdots + q_n a_{nn} = c_n \end{cases} \qquad (27)$$

The solution to this equation (27) is:

$$q_1 = \frac{\Delta q_1}{\Delta},\ q_2 = \frac{\Delta q_2}{\Delta},\ \cdots,\ q_n = \frac{\Delta q_n}{\Delta}, \qquad (28)$$

where

$$\Delta = \begin{vmatrix} a_{11}\ a_{12} \ldots a_{1n} \\ a_{21}\ a_{22} \ldots a_{2n} \\ \ldots\ldots\ \ldots\ldots\ldots \\ a_{n1}\ a_{n2} \ldots a_{nm} \end{vmatrix} \qquad (29)$$

To draw an isobar and a streamline, we write the potential at an arbitrary point M of the reservoir in the following form:

$$\Phi_M = \frac{1}{2\pi h}(q_1 \ln r_{M1} + q_2 \ln r_{M2} + \cdots + q_n \ln r_{Mn}) \qquad (30)$$

By subtracting the left and right parts of (22) and (23), respectively, we can obtain the following formula:

$$\Phi_K - \Phi_M = \frac{1}{2\pi h}\left(q_1 \ln \frac{r_K}{r_{M1}} + q_2 \ln \frac{r_K}{r_{M2}} + \cdots + q_n \ln \frac{r_K}{r_{Mn}}\right) \qquad (31)$$

Or given $\Phi_K = H(P_K)$, $\Phi_M = H(P_M)$ in (31) we get: (32)

$$H(P_M) = H(P_K) - \\ -\frac{1}{2\pi h}\left(q_1 \ln \frac{r_K}{r_{M1}} + q_2 \ln \frac{r_K}{r_{M2}} + \cdots + q_n \ln \frac{r_K}{r_{Mn}}\right) \qquad (33)$$

Since the value of pressures is the same on the isobaric lines, the values of the function H are the same at these points.

According to formula (33), the points located on the isobars are determined and constructed. Furthermore, along the orthogonal lines to the isobars, one can also determine the directions of the current lines and construct these lines.

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5. Application, results and discussion

The presented above mathematical model has been applied on a computer. A computer simulator of the development process of a gas condensate reservoir with wells of an arbitrary number was created, where the PVT properties of the hydrocarbon mixture, two-phase flow and mass transfer between phases, the change in permeability and porosity during rock compaction is taken into account. The developed simulator allows you to set the operating mode for each well separately, i.e. wells can be operated either at a given rate of production, or at a given pressure drop, or at a given bottom hole pressure. The software has been created on the basis of the following algorithm:

1. Input of PVT production data of the reservoir under consideration, reservoirs parameters and well parameters (e.g. coordinates, lift radius, operating conditions
2. Initialization of variables: E.g. T:=0; p:=p_0; s:=s_0; Q_g:=0; Q_0:=0; Δt:=τ; .
3. The initial reserves of hydrocarbons (V_g, V_c) are calculated according to (18) and (19);
4. For the considered hydrocarbon mixture, the relationship between H and p is determined by $H = \int_0^{p_0} \varphi_g(p,s)dp$ in the range [0, p_0] with a fairly small step Δp. As a result, a table is created in memory, consisting of two columns and $\frac{p_0 - 0}{\Delta p}$ rows. In the later steps, it will be used to determine the pressure at each point from the calculated value of the H pseudo pressure.
5. Saturation values s are calculated for all pressures in steps of Δp according to (17). The results are stored in memory in the form of a table consisting of columns p and s.
6. Calculation of production rates of each well (q_1, q_2, ..., q_n) according to (25).
7. Calculation of $H(p_M)$ at each point according to (33).
8. Using the dependence of $p(H)$, we pass from pseudo pressure to real pressure at each point.
9. Saturation is determined at each point using the $s(p)$ dependence
10. Production per time step of wells for gas and condensate are accumulated, respectively:
$$Q_g := Q_g + \sum_{i=1}^{n} q_{gi} \cdot \Delta t; Q_o := Q_o + \sum_{i=1}^{n} q_{oi} \cdot \Delta t$$
11. The current values of the gas and condensate recovery factors are calculated:
$$GRF = \frac{Q_g}{V_g}, CRF = \frac{Q_c}{V_c}$$
12. The time counter is being updated: T:= T+Δt.
13. The current values of pressure and saturation on the reservoir external boundaries are calculated according to (12) - (14), (16) and (17).
14. Checking the simulation termination condition - if the condition is satisfied, go to the 15th step, otherwise go to step 6.
15. Save the results.
16. Terminate the program.

The user interface of the program is shown in figure 2. Simulation results are shown in figures 3-5. Figute 3 shows the main panel of the program. It consists of three tabs- Reservoir, Wells and Simulation. The Reservoir tab provides input for the key reservoir parameters. The Wells tab includes the list of the wells with their parameters such as well type (i.e. production well, injection well, disabled well, etc.), well radius, operation condition (depression or bottomhole pressure or production rate is given), etc. The Simulation tab provides simulation options, e.g. time discretization, simulation finishing condition.

Using the simulator, some computer studies were carried out for a hypothetical reservoir. They are based on the PVT data of the gas condensate mixture of horizon VII of the Bulla-Deniz field (Azerbaijan). The following initial data were used:

Equivalent deposit radius R_e=1000 m;

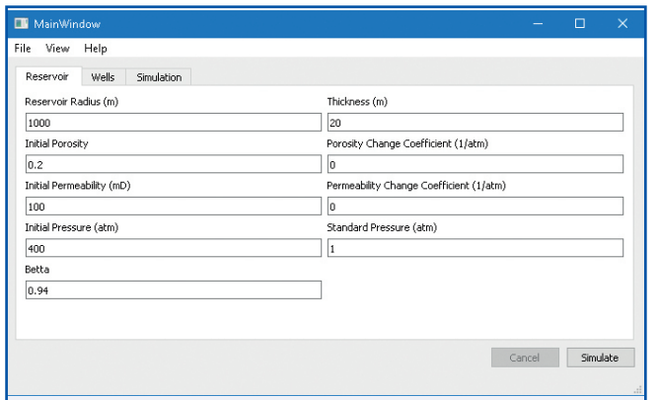

Fig. 2. Main window of the software

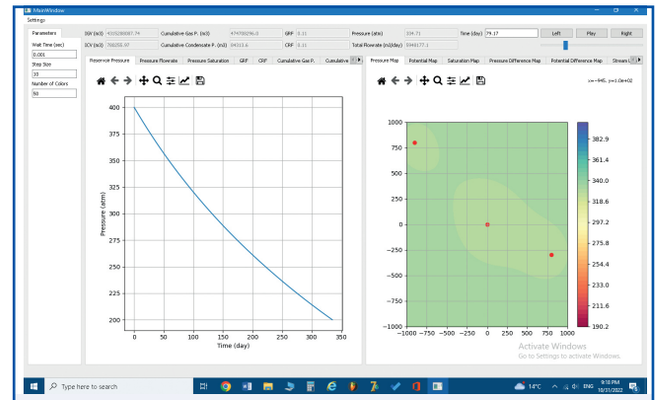

Fig. 3. Reservoir pressure dynamics curve and a map of the current pressure distribution over the reservoir

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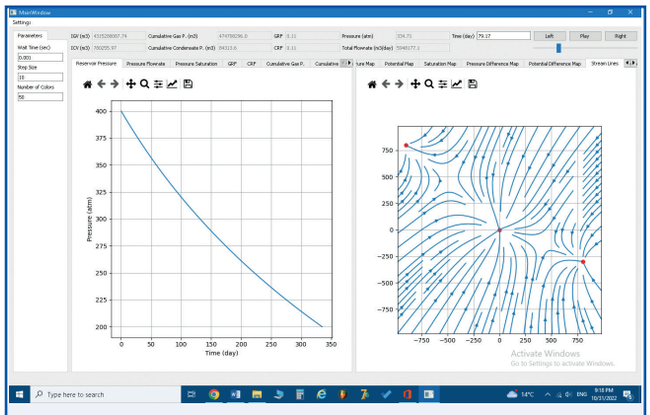

Fig. 4. Reservoir pressure dynamics curve and a map of the stream lines

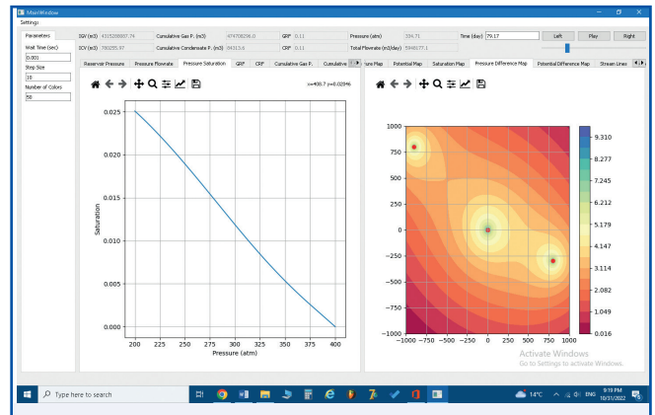

Fig. 5. Pore saturation vs pressure curve and a map of the current pressure-difference distribution over the reservoir

Initial reservoir pressure p_0=400 atm;
Initial permeability k_0=100 mD;
Initial porosity ϕ_0=0.2;
Reservoir thickness h=20 m.

The reservoir is recovered by three wells. All wells have 0.1 m radius and are operated at a given drawdown Δp=20 atm.

The results obtained (see: in fig. 3-5) showed the adequacy of the proposed approach. To verify the algorithm, the results of the development simulation for one central well were compared with the results of the semi-analytical solution obtained by [5]. The results confirmed the correctness of the idea underlying the proposed approach.

Conclusion

The application of the Binary Modeling principles in combination with some mathematical techniques with Streamline technology made it possible to simulate the development of a gas condensate reservoir. The results obtained demonstrated the wide possibilities of the proposed approach, which takes into account the PVT properties of the hydrocarbon system, two-phase flow, phase transformation, mass transfer between phases, and compressibility of reservoir rocks. The proposed hybrid approach can also be applied to volatile oil reservoirs. The obtained results once again showed the wide possibilities of the Streamline method based on the theory of potential flow.

Nomenclature
Parameters with the «o» and «g» index correspond to a liquid and gas phase, respectively;
p_0= initial reservoir pressure, atm
p_w= bottom-hole pressure, atm
p_e= pressure at the external reservoir boundary, atm
p_{at}= atmospheric pressure, atm
r_e= reservoir or well drainage area radius, m
r_w= wellbore radius, m
a_m= rock compressibility factor, 1/ atm
k= formation permeability, 10^{-12} m^2
k_0= initial permeability, 10^{-12} m^2
k_{ro}= liquid-phase relative permeability
k_{rg}= gas-phase relative permeability
s= liquid (condensate) saturation
v= velocity, m/s
Ω= porous volume, m^3
Ω_0= initial porous volume, m^3
q_o= condensate production rate, m^3/s
μ_o= condensate viscosity, atm·s
μ_g= gas-phase viscosity, atm·s

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ρ = condensate density
B_o = condensate formation volume factor
R_s = solubility of gas in liquid phase, m^3/m^3
z = Z-factor
β = temperature correction factor
c = vaporous hydrocarbons content of the gas phase, m^3/m^3
$\bar{\gamma} = \frac{\gamma_o(p)}{\gamma_g(p)}$ = ratio of the specific weight of liquid phase and the specific weight of gas phase at reservoir pressure;
ϕ = formation porosity
ϕ_0 = initial formation porosity
t = time, s

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