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STATISTICAL MODELING OF OIL RESERVOIR LIFE CYCLE

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ABSTRACT
Based on statistical analysis, a method of stage-by-stage structuring and allocation of the boundaries of the stages of oil field development is proposed. The procedures for calculating the remaining recoverable reserves and the time to reach the maximum level of current oil production are given. Based on the values of accumulated oil production at the final stage of development, using the adaptive Kalman filter, forecasts of oil production are constructed in discrete time and estimates of the possibility of achieving design indicators with the existing development system are given.

KEYWORDS:
Stage-to-stage structuring;
Development stages;
Forecast of recoverable reserves;
Predicted oil production;
Kalman filter.

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Introduction

The concept of the stage and the criteria for selection oil fields development stages were formulated in the mid 70s of last century. The paper [1] recommends dividing oil production dynamics into four successive stages and presents some approaches to identification the boundaries between the development stages. However, these approaches are based on heuristic evaluations with no well-defined criteria. Another methodological approach proposed in [2], uses the design value of the initial recoverable reserves (IRR), which is usually significantly adjusted during field development. The paper [3] considers the possibility of oil recovery description by analogy with the branching-chain reaction dynamics for detection of development stage boundaries. Weibull distribution widely used in the theory of reliability is a statistical analogue of Kolmogorov – Erofeeva's kinetic equation used here. This distribution, shown below, is a good approximation of a fourth stage of field development.

Furthermore, Kolmogorov-Erofeev's equation used in the work [3] comes to the two-parameter Weibull distribution, which is only applied in case when the shift parameter δ (or «minimal run» in the terminology of the theory of reliability) is equal to zero. In general, Weibull distribution is a three-parameter one with $\delta \geq 0$, defined on a given sample. The third stage, when there appears water cut resulted in oil production decline, should be well described by a Pareto distribution, having a heavy right tail and reflecting hyperbolic decline in oil production.

Problems of decline curves analysis, discussed in the papers [4-7], apply only to the III and IV stages of the cycle. Empirical equations proposed in the above-mentioned works give the typical decline curves only in case of unique solvability. Otherwise, the question remains undecided.

We should note the variety of solving partitioning field development life cycle into the individual stages as well as the variety of their titles which exists in the literature. For example,[8] consider four stages (phases) of development: developing, plateau, decline and mature and using dynamic counterparts of previous developments define the basic characteristics of field exploitation: the cumulative oil production, the ratio of produced oil and water, etc., and find prediction estimates of oil production and the remaining recoverable reserves. [9] divide the process of hydrocarbon deposits development into three phases: build-up, plateau and decline, and the fourth phase - decommissioning is still considered technically feasible for operation, though unprofitable from a financial point of view, and therefore recommend transferring the field to reserve in this phase.

By the way, the life cycle of any commercial product manufacture is also divided into four successive phases. In the paper [10], these stages are named as: introduction, grouth, maturity and decline, like in the work [11]: introduction to market, grouth, maturity and decline.

In this paper, based on statistical study the life cycle is divided into four sequential steps, conventionally named as: build-up, developing, decline and maturity. We see that the different stages of the oil fields development differ in the type of distribution of current oil production. On the basis of maximum compactness we have developed a methodical approach to the determination of the boundary points of the adjacent stages within oil production curve accuracy which help to calculate the duration of each step. The reliability of the results is determined by the compliance rate between the theoretical and empirical distribution functions.

The numerical calculation procedure of the recoverable oil reserves and the time of peak oil production have been proposed. Moreover, the possibility to reduce the Weibull distribution function values for the current oil production in the fourth development stage to solution of the logistic model for normalized cumulative oil production have been presented, allowing to build one-step and multi-step oil production predictions in the final development stage using an adaptive Kalman filter in discrete time and evaluate real opportunities to achieve design performance under the current system of development.

1. Stage-to-stage structuring of development

Entire life cycle T of oil field development can be divided into disjoint intervals (stages) Tj (j=1,...,4) in accordance with the time behavior of basic development parameter (oil production per year t) $X=q(t)$. The intervals T_1 and T_2 reflect the increase of X value with a higher speed in T_2, rather than in T_1, the interval T_3 – reflects saturation and the following sharp decrease of X value, and the interval T_4 – relative stabilization with a small decrease of its values.

In order to identify distribution functions of X value at the stages Tj (j=1,…,4) let us present the cycle $T=[t_1, t_2]$ as a set-theoretical sum of intervals $\tilde{T}_j = \left[\tilde{t}^i_j, \tilde{t}^e_j\right]$ $(j = 1,...,4)$, preserving the above mentioned behavior of X value in the intervals $\tilde{T}_j$ correspondingly. With that we will suppose that neighboring intervals $\tilde{T}_j$ and $\tilde{T}_{j'}$ cross (fig.1) and their crossing $\tilde{T}_{j,j'} = \left[\tilde{t}^i_{j,j'}, \tilde{t}^e_{j,j'}\right]$ fulfils the condition

$$\tilde{t}^i_{j,j'} = \tilde{t}^i_{j'},\ \ \tilde{t}^e_{j,j'} = \tilde{t}^e_j.\ \ (j = 1,2,3),\quad \tilde{t}^i_1 = t_1,\quad \tilde{t}^e_4 = t_2 \tag{1}$$

The distribution parameters of X value at the stage T_j will be determined based on the learning sample X^{learn}_j, composed of X values at $t \in T$

1.1. Data pre-processing

We have collected the data on 20 fields from the different regions in order to investigate the life cycle of oil field development.

Let $T_j=[t^i_j, t^e_j]$ denote the life cycle of field M_j(j=1,...,j_0) and let $\left\{q^j_{t_j}\right\}_{t_j \in T^\partial_j}$ denote the time series of annual oil production values in the field M_j, i.e. the given series of observations q^j on the discrete (with the discreteness interval Δt_j=1) set of points $T^\partial_j \subset T_j$. Let us normalize the series $\left\{q^j_{t_j}\right\}_{t_j \in T^\partial_j}$ by the largest value $q^j_{\max} = \max\limits_{t_j \in T^\partial_j} q^j$ substituting

$$\tilde{q}^j_{t_j} = q^j_{t_j} / q^j_{\max}$$

So we'll obtain the time series $\left\{\tilde{q}^j_{t_j}\right\}_{t_j \in T^\partial_j}$ for each j-th field with the values $\tilde{q}^j_{t_j}$ from the interval [0,1]. In order to align these time series to one time interval let us normalize them in time substituting

$$\tilde{t}_j = t_j / t^*_j,\quad j = 1,\cdots j_0,$$

where t^*_j – is t_j value, at which we can achieve the maximum value $\tilde{q}^j$ of the factor $\tilde{q}^j_{t_j}$, that is $t^*_j = \arg\max\limits_{t_j \in T^\partial_j} \tilde{q}^j_{t_j}$. As a result, under every fixed j we'll obtain the time series $\left\{\tilde{q}^j_{\tilde{t}_j}\right\}_{\tilde{t}_j \in \tilde{T}^\partial}$, where $\tilde{T}^\partial_j$ – reflects $\tilde{T}^\partial_j$ after the mentioned time substitution. With that the maximum value of these series equal to $\max\limits_{t_j \in T^\partial_j} \tilde{q}^j_{t_j}$, will be obtained in the point $\tilde{t}^*_j = 1$ for all j=1,..., j_0.

Figure 2 shows the field of points corresponding to the values of time series $\left\{\tilde{q}^j_{\tilde{t}_j}\right\}_{\tilde{t}_j \in \tilde{T}^\partial_j}$ $(j = 1,\cdots, j_0)$. Let us denote $\tilde{t}^i = \min\limits_{j=1,j_0} \min\limits_{\tilde{t}_j \in \tilde{T}^\partial_j} \tilde{t}_j$, $\tilde{t}^e = \max\limits_{j=1,j_0} \max\limits_{\tilde{t}_j \in \tilde{T}^\partial_j} \tilde{t}_j$, $\tilde{T} = \left[\tilde{t}^i, \tilde{t}^e\right]$.

Let $\tilde{T}^\partial$ denote the set-theoretical sum $\tilde{T}^\partial = \bigcup\limits_{j=7}^{j_0} \tilde{T}^\partial \subset \tilde{T}$. Each element of discrete set $\tilde{T}^\partial$, which in general is unequal series (non-equidistant from each other) of points, is assigned one or several values of $\tilde{q}_{\tilde{t}}$.

Hence, in the plane $(\tilde{t}, \tilde{q})$ we have the discrete set of points $\{\tilde{t}, \tilde{q}_{\tilde{t}}\}_{\tilde{t} \in \tilde{T}^\partial}$, describing oil production dynamics within the life cycle $\tilde{T}$ of a certain generalized field M. The dynamics of M field development is obtained by smoothing the above two-dimensional set of points $\{\tilde{t}, \tilde{q}_{\tilde{t}}\}_{\tilde{t} \in \tilde{T}^a}$.

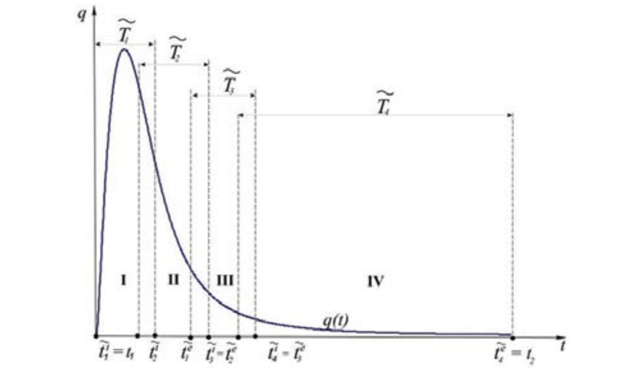

Fig.1. Dividing the life cycle into intervals $\tilde{T}_j$ $(j = 1,...,4)$

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Identification of a probability distribution of a current oil production value $\tilde{q}_{\tilde{t}}$ will be carried out using standard distributions $F(x)$ that are differentiable functions. Therefore, $\tilde{q}_{\tilde{t}}$ needs preliminary approximation with some smooth function. For this purpose, we use Savitzky–Golay smoothing filter, in short SG-filter [12].

The application of SG- filter over the set of points $\{\tilde{t}, \tilde{q}_{\tilde{t}}\}_{\tilde{t}\in\tilde{T}^{\partial}}$ provides the best piecewise-polynomial smoothing with the 4-th order polynoms with a sliding data review window having the length of $2m+1$ with $m=8$ (fig.3).

The same approximation quality we obtain when use the nonlinear least squares method which ends in a fractional rational function dependence (fig.3)

$$y=(a+cx+ex^2+gx^3+ix^4+kx^5)/(1+bx+dx^2+fx^3+hx^4+jx^5)$$

with coefficients

a=0.034203417, b=-3.2323297, c=0.40983017, d=4.5374077, e=-1.0344214, f=-3.4316344, g=0.67941989, h=1.1681308, i=-0.08666884, j=-0.035657196, k=0.003246577.

With that, the coefficient of multiple determination R^2=0.944, which proves a close correlation of clouds in figure 2.

SG-filter provides the digitalization $(t_i, q_i)_{i=\overline{i,N}}$ of smoothed oil production values q in the generalized field M. With that case the values t_i are determined as the left ends of sliding given data review window. To simplify the calculations from the above subsequence we'll distinguish a partial sequence $\{t_{i_k}, q_{i_k}\}_{k=\overline{1,k_0}}$, where $i_k = 1+5(k-1)$, $k=\overline{1,k_0}$; $1+5(k_0-1)\le N < 1+5k_0$.

Let us divide the subsequence $\{t_{i_k}\}_{k=\overline{1,k_0}}$ into 4 crossing partial sequences $\tilde{T}_1^{об},\cdots,\tilde{T}_4^{об}$ that fulfill the condition (1), supposing that the life cycle T=[t_1, t_2], where $t_1 = \min\limits_{k=\overline{1,k_0}} t_{i_k}$, $t_2 = \max\limits_{k=\overline{1,k_0}} t_{i_k}$. It is obvious that $\tilde{T}^{learn} = \bigcup\limits_{j=1}^{4} \tilde{T}_j^{learn}$ – is a discrete subset from T. Let X_j^{learn} denote the set of values q, corresponding to t *from* $\tilde{T}_j^{learn}$. The learning samples ^{learn} will be used below to identify the distribution of random value X (annual oil production in M field) in

$$X_j = \left(x_j^i, x_j^e\right), \ where\ x_j^i = \min_{x\in x_j^{learn}}, \ x_j^e = \max_{x\in X_j^{learn}}; \ X_j^{learn} \subset X_j.$$

1.2. Numerical implementation of the method

Let us divide the whole life cycle T=[0.027; 14] into the learning samples $\tilde{T}_j^{learn} = \left[\tilde{t}_j^{\,i}, \tilde{t}_j^{\,e}\right]$, where $\tilde{t}_1^{\,i} = 0.027$, $t_1^e = 0.648$; $\tilde{t}_2^{\,i} = 0.544$, $\tilde{t}_2^{\,e} = 1.131$; $\tilde{t}_3^{\,i} = 1.062$, $\tilde{t}_3^{\,e} = 4.477$, $\tilde{t}_4^{\,i} = 4.063$, $\tilde{t}_4^{\,e} = 14$. Learning samples X_j^{learn} of x values conform to intervals $\tilde{T}_j^{learn}$.

1.2.1 Stage-to-stage structuring

Stage 1

Estimation of distribution parameters

Let us substitute z=lg$x$ for the values x of X_1 and advance a hypothesis that the random value X is distributed on X_1 by logarithmically normal law with the distribution function $F_0(x) = \Phi\left(\frac{\lg x - \lg x_0}{\sigma_z}\right)$ where $\Phi(z)$ – is distribution function of standard normal random value (with zero mean and single dispersion).

With regard to specified substitution of a variable we obtain: $\lg x_0 = z_0 \approx \bar{z}$, $\bar{z} = \frac{1}{n}\sum\limits_{i=1}^{n} z_i$, z_i=lgx_i, (n=19), σ=σ_z≈s_z, $s_z = \left\{\frac{1}{n-1}\sum\limits_{i=1}^{n}(z_i - \bar{z})^2\right\}^{½}$.

If we insert the values of x_i from X_j^{learn} in these formulas we find z_0=-0.740, σ_z=0.483.

If the suggested hypothesis is true, then X is

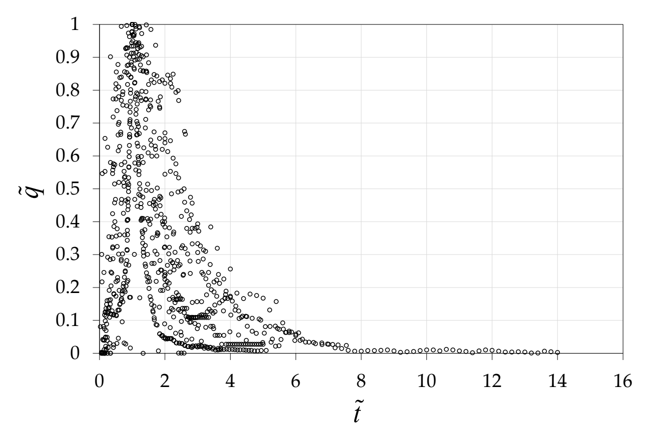

Fig.2. M_j(j=1,..., j_0) The dynamics of fields' development in normalized coordinates

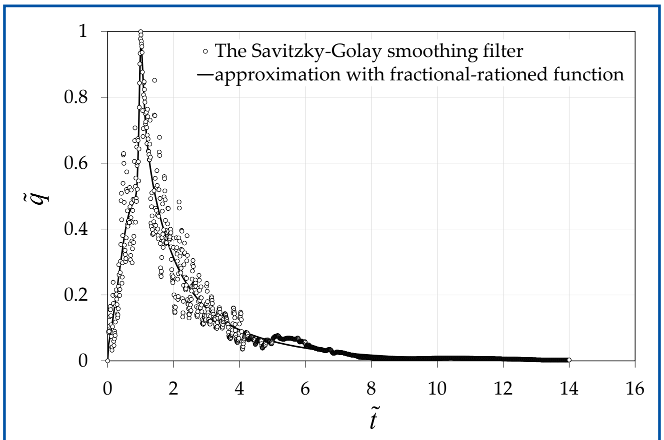

Fig.3. The dynamics of generalized field development smoothed with The Savitzky–Golay smoothing filter

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distributed in X_1 by logarithmically normal law with the distribution function

$$F_0(x) = \Phi\left(\frac{\lg x - \lg x_0}{\sigma_z}\right)$$

Goodness-of-fit test

At each stage of development the goodness-of-fit test of the selected hypothetical distribution function $F_0(x)$ versus the truth distribution function $F(x)$, i.e. proof of hypothesis $H_0: F(x)=F_0(x)$, will be realized owing to Kolmogorov goodness-of-fit test.

Let $x_1,\ldots, x_n$ – is the sample consisting of n independent observations where n_1 ($n_1 \le n$) values differ. Ordering them increasingly we obtain a set of variate values

$$x_{(1)} < x_{(2)} < \cdots < x_{(i)} < \cdots < x_{(n_1)}$$

Let the value $x_{(i)}$ is k_i times repeated in the original sample $x_1,\ldots, x_n$. Let us construct the empirical distribution function $\hat{F}_n(x)$:

$$\hat{F}_n(x) = \begin{cases} 0, & -\infty < x < x_{(1)}, \\ (i + k_i - 1)/n, & x_{(i)} \le x \angle x_{(i+1)}, \quad i = 1,\ldots,n_1 - 1, \\ 1, & x_{(n_1)} \le x \angle \infty . \end{cases} \quad (2)$$

Kolmogorov statistic is $D_n = \sup_x \left| \hat{F}_n(x) - F_0(x) \right|$.

We construct a variation series of x values of X_1^{learn} in order to check the hypothesis on logarithmically normal distribution of a random value X on the set X_1=[0.019; 0.448], and we calculate the value $D_{(i)} = \max_{x \in \Delta_{(i)}} \left|\hat{F}_n(x) - F_0(x)\right| = \left|\hat{F}_n(x_{(i)}) - F_0(x_{(i)})\right|$ at each interval $\Delta_{(i)} = \{x : x_{(i)} \le x < x_{(i+1)}\}$ where $\hat{F}_n(x)$ is calculated by formula (2). The calculated value D_n^c of D_n statistic is found of the relationship

$$D_n^c = \max_{i=1,n_1} D_{(i)} \quad (3)$$

For X_1^{learn} with n=19 by formula (3) we will obtain $D_n^c = 0.18$. The critical value for Kolmogorov criterion of goodness if n=19 and α=0.20, is equal to 0.237. As $D_n^c < 0.237$ the hypothesis $H_0: F(x) = F_0(x)$, if $x \in X_1$, can be accepted with the confidence probability P_1=0.8

Similarly, the distribution functions are identified for the remaining stages. The calculation results of distribution functions and confidence probability of fitting criterion for all four stages are shown in table 1. These confidence probabilities can be improved by reducing the observation scale.

We should note at each stage of the identification of the real distribution function $F(x)$ several competing distributions were considered as a theoretical distribution function $F_0(x)$: normal, logarithmically normal, Exponential, Pareto and Weibull distribution. Preference was given to the distribution law, when with the greatest confidence level could be accepted the hypothesis $H: F(t) = F_0(t)$ by the Kolmogorov goodness-of-fit criterion.

So, the entire oil field life cycle can be divided into four sequential stages: build-up, developing, decline and maturity, which are reliably (80% at least), described to be respective logarithmically normal, exponential, Pareto and Weibull distributions.

The first period (build-up) – is the initial stage of development with the unsteady regime and the adaptive selection of production engineering in accordance with the filtration properties of

Table 1

The calculation results of distribution functions and confidence probability of fitting criterion for all four stages

Phases	Distribution law	Distribution function	Probability belief of fitting criterion
I	logarithmically normal	$F_0(x) = \Phi\left(\frac{\lg x + 0.740}{0.483}\right), \quad x > 0$	$P_1 = 0.8$
II	exponential	$F_0(x) = \begin{cases} 1 - \exp\{-4.861(x - 0.388)\}, & x \ge 0.388 \\ 0, & x < 0.388 \end{cases}$	$P_2 = 0.8$
III	Pareto	$F_0(x) = \begin{cases} 1 - (0.254/x)^{3.181}, & x \ge 0.254 \\ 0, & x < 0.254 \end{cases}$	$P_3 = 0.99$
IV	Weibull	$F_0(x) = \begin{cases} 1 - \exp\left\{-\frac{(x - 0.003)^{1.104}}{0.011}\right\}, & x \ge 0.003 \\ 0, & x < 0.003 \end{cases}$	$P_4 = 0.85$

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the reservoir. The second period (developing) is associated with an increase of percent of oil recovery from the reservoir and continues until the maximum level of oil production and the possible stabilization of this level as saturation level (the so-called «plateau»). The third period (decline) exhibits a sharp decline in oil production due to flooding and formation contamination. Finally, the fourth period (maturity) reflects the comprehensive application of natural operation methods with a fluctuating relative to some minimum level of oil production, which requires additional (artificial) methods of enhanced oil recovery.

1.2.2. Determination of adjacent stages' limits

In accordance with boundary detection method between adjacent stages, described in Appendix A, we check the intersection possibility of graphs $f_1(x)$ and $f_2(x)$, $f_2(x)$ and $f_3(x)$, $f_3(x)$ and $f_4(x)$, where $f_1(x)$, $f_2(x)$, $f_3(x)$ and $f_4(x)$ – are the probability density functions corresponding to stages I, II, III and IV

According to the above method of detection of stages neighbouring boundaries we check the possibility of $f_1(x)$ and $f_2(x)$, $f_2(x)$ and $f_3(x)$, $f_3(x)$ and $f_4(x)$ plots crossing.

We have A=0.027 and B=14 for the field M. The analysis of current oil production frequency curves showed that $f_1(x)$ disjoints $f_2(x)$ in the set $G_{1,2}=[0;\infty)\cap[0.388;\infty)\cap[A,B]=[0.388;14)$;

$f_2(x)$ disjoints $f_3(x)$ in the set

$G_{2,3}=[0.388;\infty)\cap[0.254;\infty)\cap[A,B]=[0.388;14)$

and finally $f_3(x)$ disjoints $f_4(x)$ in the set $G_{3,4}=[0.254;\infty)\cap[0.004;\infty)\cap[A,B]=[0.254;14)$. Then by formula (4) we obtain

$x^{1,2}_{bound}=\max\{0;0.388\}=0.388;$

$x^{2,3}_{bound}=\max\{0.388;0.254\}=0.388;$

$x^{3,4}_{bound}=\max\{0.254;0.004\}=0.254.$

As $\tilde{T}_{1,2}=[0.544;0.648]$, $\tilde{T}_{2,3}=[1.062;1.131]$ and $\tilde{T}_{3,4}=[4.063;4.477]$, by formula (3) we find

$t^{1,2}_{bound}=0.544,\quad \hat{t}^{2,3}_{bound}=1.131,\quad \hat{t}^{3,4}_{bound}=4.063.$

2. The determination of initial recoverable reserves and peak level of current oil production

Let $Q(t)=\sum_{t'\le t} q(t')$ – denote cumulative oil production at the moment t. As current oil production is relatively stable at the closing stage, there exists a finite limit $Q_\infty=\lim_{t\to\infty} Q(t)$ equal to the initial recoverable reserves. Below we present the procedures to determine the value Q_∞ and the peak level q_{max} of current oil production.

For calculation of value Q_∞ we propose to use the linear regression $g=\alpha_0+\alpha_1 x$, where $x=Q$, and $y=q/Q$, q and Q – is a current and cumulative oil production. The coefficients q and Q are obtained by the least-squares method (LSM) on the basis of $\{Q_i,(q/Q)_i\}$ observations at the moments t_i (i=1,..., n) on IV stage of the life cycle of field development. According to the LSM estimates we calculate $\hat{\alpha}_0$ and $\hat{\alpha}_1$

$$Q_\infty=\hat{\alpha}_0/\hat{\alpha}_1 \tag{4}$$

Detailed proof of formula (4) is given in Appendix B.

For estimation of the peak level of current oil production $q(t)$ we find a critical function point q (let us denote it t_{peak}), which is equivalent to finding the flex point of autocatalytic curve $Q(t)$, where – $Q(t)$ is the cumulative oil production per year.

The computational procedure for assessing the value t_{peak} and the calculated values t_{peak} and are $q(t_{peak})$ listed in Appendix B.

It should be noted that initial recoverable reserves Q_∞ can be calculated only in the IV stage, whereas a maximum level of current oil production $q(t_{peak})$ in the II stage of lifetime of field development.

3. Predicted oil production

As shown above, the probabilities distribution function of values x of the random value $X=q(t)$ at fourth stage of development (that is if $\tilde{t}\in\tilde{T}$) is well described by Weibull distribution

$$F(x)=1-\exp\left\{-\frac{(x-\delta)^\beta}{\theta}\right\},$$

where δ=0.003, β=1.104, θ=0.011

As we have the discrete x_i values of the random value X, then $F(x)=P(X\le x\}=\sum_{i/x_i\le x} p_i$, where $p_i=P(x=q_i=q(\tilde{t}_i))$. Then $p_i=q_i/Q_\infty$ due to geometric probability and, it follows that $F(x|_{x=q(\tilde{t})})=Q(\tilde{t})/Q_\infty$.

Let $\tilde{t}_0$ - be such a point from the interval $\tilde{T}_4=[4.063;14]$ that the interval $[\tilde{t}_0;14]$ contains at least n(n≥50) measurements $x(\tilde{t})=q(\tilde{t})$. Let formulate function $\tilde{x}(\tilde{t})$, obtained from $x(t)$ following substitution $\tilde{x}(\tilde{t})=x(\tilde{t}+t_0)-\delta$ (δ=0.003 - is the F4(x)function parameter), as a scaling relationship $\tilde{x}(\tilde{t})=A/\tilde{t}^{\mu}$, $\mu>0, \tilde{t}\in[\tilde{t}_1,\tilde{t}_2]$, $\tilde{t}_1=0, \tilde{t}_2=14-\tilde{t}_0$. The parameters A and μ of this relation are defined through regression $Y=\alpha_0+\alpha_1 X$, where $Y=\ell n\,\tilde{x}$, $X=\ell n\,\tilde{t}$, $\alpha_0=\ell n\,A$, $\alpha_1=-\mu$. We estimate $\hat{\alpha}_0=-6.706$, $\hat{\alpha}_1=-0.128$ with the aid of LSM, whence we obtain assessed values $\hat{A}=e^{\hat{\alpha}_0}$, $\hat{\mu}=-\hat{\alpha}_1$.

After the above substitutions with regard to scaling relationship the distribution function $F_4(x)$ of values $x(\tilde{t})=q(\tilde{t})$ when $\tilde{t}\in[\tilde{t}_1,\tilde{t}_2]$ is written as $F_4(\tilde{t})=1-\exp\{-b\cdot\tilde{t}^{a}\}$, $b=0.0555$, $a=0.128$. Then $\lambda(\tilde{t})=F_4'(\tilde{t})=ab\tilde{t}^{a-1}=0.007/\tilde{t}^{0.872}$. The cumulative normalized oil production $Q_a(\tilde{t})=Q(\tilde{t})/Q_\infty$ satisfies the equation

$$\frac{dQ_{cn}(\tilde{t})}{d\tilde{t}} = \lambda(\tilde{t})(1 - Q_{cn}(\tilde{t})), \quad \tilde{t} \in [\tilde{t}_1, \tilde{t}_2], \tag{5}$$

where $\lambda(\tilde{t}) = ab\tilde{t}^{a-1}$ – is oil reserves production rate.

The equation (10) is used in (3) to describe oil production dynamics and is the analogue of Kolmogorov-Erofeyev equation for chain branch chemical reactions. For cumulative oil production $Q(\tilde{t})$ from (5) we obtain the following equation

$$\frac{dQ(\tilde{t})}{d\tilde{t}} = -\lambda(\tilde{t})Q(\tilde{t}) + \lambda(\tilde{t})Q_{\infty} + w(\tilde{t}) \ , \tag{6}$$

where $w(\tilde{t})$ – is admissible random disturbance affecting the process state. If, moreover, the state $x(\tilde{t}) = Q(\tilde{t})$ is measured by the value

$$y(\tilde{t}) = C(\tilde{t})x(\tilde{t}) + \upsilon(\tilde{t}), \tag{7}$$

where $\upsilon(\tilde{t})$ – is the measurement noise, then equations (6) – (7) describe the linear dynamic system in continuous time.

The key points of a suggested statistical simulation method of oil field development lifetime are the following:

a) Partition of lifetime into four consecutive stages: build-up, developing, decline and maturity;

b) Identifying a probability distribution of a current oil production value $q(t)$ by virtue of standard distributions: log-normal, exponential, Pareto and Weibull distributions;

c) Alternation of probability distributions laws of a value of quantity $q(t)$ testifies transition to a subsequent stage of development;

d) A determination method of boundary points of adjacent stages has been suggested and this method characterizes duration of a previous stage;

e) Calculating formulas have been given for a maximum production level and initial recoverable reserves by values $q(t)$ for II and IV stages accordingly;

f) Forecast models have been developed for cumulative oil production $Q(t)$. These models enable assessing a possible level of reaching designed production economics in the near future and correcting a strategy of further field development on the basis of comparison of designed and predictable recoverable reserves.

4. Application example

For illustration of a suggested statistical simulation method of lifetime, we will consider an analysis example of a development condition of one specific horizon of Uzen field (Kazakhstan).

Let us consider the current oil production history from the start of «Zhetybay» field production (fig.4). (conventional production values have been presented)

As is clear from figure 4 current oil production history is visually divisible into two parts – 1965 - 1999 and since 1999 up to date. It is due to the fact that up to 1999 a part of the field was on-stream, whereas the remaining part of the field was drill out after 1999.

Separation of development stage over a period of 1965-1999

The calculation data of distribution function and goodness-of-fit test probability belief [2] for all the four stages are listed in the table 2.

Thus, the entire life cycle of the field is possible to divide onto four consecutive steps: build-up, developing, decline and maturity , which with plenty

Table 2

The calculation data of distribution function and goodness-of-fit test probability belief for all the four stages

Stages	Distribution law	Distribution function	Probability belief of goodness-of-fit test
I	logarithmically normal	$F_0(x) = \Phi\left(\frac{\lg x - 1.823}{0.451}\right), x > 0$	0.9
II	exponential	$F_0(x) = \begin{cases} 1 - \exp\{-0.0102(x - 146.09)\}, & x \ge 146.09 \\ 0 & , x < 146.09 \end{cases}$	0.9995
III	Pareto	$F_0(x) = \begin{cases} 1 - (44.765(x))^{0.777}, & x \ge 44.765 \\ 0 & , x < 44.765 \end{cases}$	0.8
IV	Weibull	$F_0(x) = \begin{cases} 1 - \exp\left\{\frac{-(x - 44.76)^{2.67}}{134.928}\right\}, & x \ge 44.76 \\ 0 & , x < 44.76 \end{cases}$	0.8

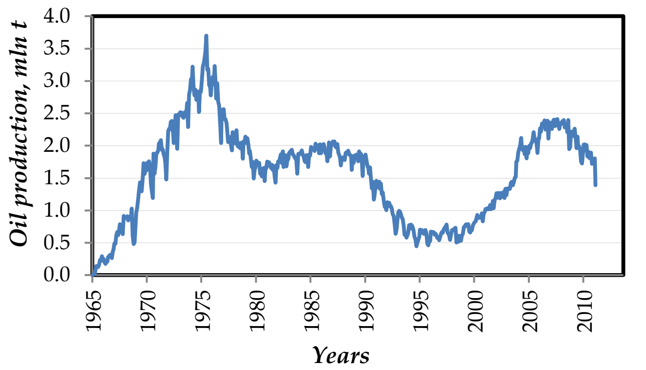

Fig.4. The current oil production history

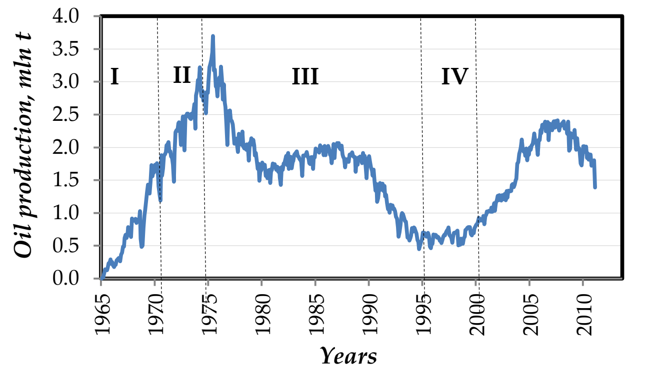

Fig.5. Separation of field development stages over a period of 1965-1999

good enough accuracy are described by respectively logarithmically normal, exponential, Pareto and Weibull distributions: T_1=[01.1965; 01.1972], T_2=[01.1972; 10.1975], T_3=[10.1975; 01.1995], T_4=[01.1995; 12.1999] (fig.5). Reference point t=0 in figure 5 corresponds to commencement date of development - 01.1965.

Evaluation of initial recoverable reserves

Distribution function for value $X=q(t)$ ($q(t)$ - current oil production) on IV stage can be written in the form:

$$F_4(x) = 1 - \exp\left\{ -\frac{(x-\delta)^{\beta}}{\theta} \right\}, \quad x \geq \delta,$$

where δ=44.76, β=2.67, θ=134,925.

Subsequent to substitution $\tilde{x}(\tilde{t}) = x(\tilde{t} + t_0) - \delta$, t_0=29 (which is equivalent of date 01.1995) let's represent function $\tilde{x}(\tilde{t})$ as a scaling relationship

$$\tilde{x}(t) = A / \tilde{t}^{\mu}.$$

Let's construct a regressional relationship $Y = \ln \tilde{x}$ from $X = \ln \tilde{t}\,(\tilde{t} = t - t_0)$ by points $t \in T_4 = [01.1995;12.1999]$ t(which in order provisional scale corresponds to interval [29, 34])

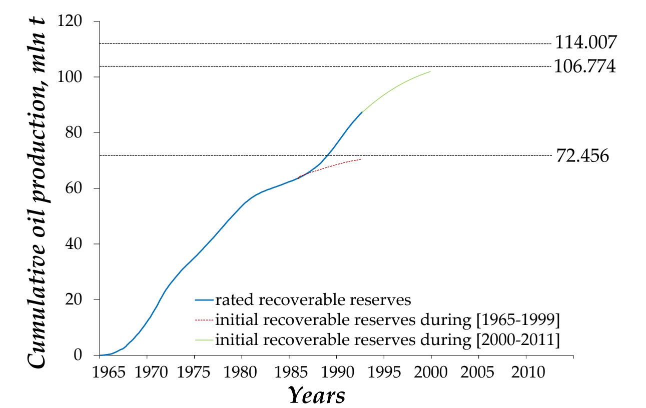

Fig.6. Rated and forecast data of initial recoverable reserves during [1965-1999] and [2000-2011]

$$Y = \alpha_0 + \alpha_1 X$$

LS method - estimate of parameters α_0 и α_1;

$$\hat{\alpha}_0 = 4.02, \qquad \hat{\alpha}_1 = 0.05$$

Whereof we obtain estimate of parameters A and μ;

$$\hat{A} = e^{\hat{\alpha}_0} = 55.496, \ \hat{\mu} = -\alpha_1 = -0.05$$

Subsequent to above substitutions, with regard to scaling relationship, we obtain

$$F_4(\tilde{t}) = 1 - \exp\{-b\tilde{t}^{a}\}, \ b\text{=48.122} \quad a\text{=0.036}.$$

Then

$$F_4^1(\tilde{t}) = ab\tilde{t}^{a-1}\exp\{-b\tilde{t}^{a}\} = 1.754\tilde{t}^{0.964}\exp\{-48.122\tilde{t}^{0.036}\}$$

Cumulative standard oil production $Q_{HH}(\tilde{t})$ $Q(\tilde{t})/Q_{\infty}$ meets the difference equation

$$\frac{dQ_{HH}(\tilde{t})}{d\tilde{t}} = \lambda(\tilde{t})(1 - Q_{HH}(\tilde{t})), \quad \tilde{t} \in [\tilde{t}_1, \tilde{t}_2],$$

where $\lambda(\tilde{t}) = ab\tilde{t}^{a-1}$ - is oil reserves production rate: $\tilde{t}_1 = 0$, $\tilde{t}_2$ =34-29=5.

Going to discrete values (in increments of discreteness τ) x_k=$Q(k\tau)$, λ_k=$\lambda(k\tau)$, A_K=(1-$\tau\lambda_K$), UK=$\tau\lambda_K Q_{\infty}$ from differential equation we obtain difference equation system:

$$\begin{cases} x_{k=1} = A_K x_k + U_K + \tau w_k \\ y_k = C_K x_k + v_k \end{cases}$$

where w_k and v_k – are noise process status and experimental observation troubles respectively.

The first of the simultaneous equations is an equation of state, the second one is an experimental observation equation (in the calculations has been accepted C_K≡1)

Using the Kalman algorithm of optimal linear filtering with discrete time periods we obtain the forecasts $Q(t)$ in the period [1965-1999] and [2000-2011].

As is clear from figure 6 when forecast in the period [1965-1999] the initial reserves composed 72.5 mln tones, i.e. total 63% of rated initial recoverable reserves (3D geological field model based estimated original oil in place), whereas subsequent to drilling-out operation of the deposit since 2000 the initial recoverable reserves under the forecast in the period [2000-2011] composed 106.8 mln tones, which makes 94% of rated initial recoverable reserves.

Further stabilization, oil production enhancement and cumulative gain in recoverable reserves in the field require application of state-of-the-art bottom-hole zone stimulation methods. Hence, the proposed approach enables to evaluate the efficiency of the strategy used to plan and develop activities on enhancement the level of efficiency achieved.

Conclusions

- Based on the identification of current oil production values distribution at the separate stages we propose the statistic method to detect boundary points of the neighbouring stages and determine the durability of each stage of the oil field development. With that we show that the distribution of current oil production values at the consecutive stages I-IV with the sufficiently high confidence is approximated according to lognormal, exponential, Pareto and Weibull distributions.
- Availability of every consecutive stage duration enables defining a maximum level of current oil production in the II stage and calculating initial recoverable oil reserves in the IV stage under a current field development strategy.
- We have shown that Weibull distribution function of current oil production values at the fourth stage of the life cycle development, relying on the *q-t* scaling dependence, aligns with the linear differential equation solution relatively to the normalized cumulative oil production that is the analogue of Kolmogorov-Erofeyev kinetic equation.
- With the help of Kalman adaptive filter we have obtained the calculation formulas for the one-step and multistep oil production prediction at the closing stage of development with the estimation of the prediction error and its variance.

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Статистическое моделирование жизненного цикла разработки нефтяного месторождения

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Реферат

На основе статистического анализа предложена методика постадийной структуризации и выделения границ стадий разработки нефтяного месторождения. Приведены процедуры расчета остаточных извлекаемых запасов и времени достижения максимального уровня текущей добычи нефти. По значениям накопленной добычи нефти на завершающей стадии разработки с помощью адаптивного фильтра Калмана в дискретном времени построены прогнозы добычи нефти и даны оценки возможности достижения проектных показателей при существующей системе разработки.

***Ключевые слова*:** постадийная структуризация; выделение границ стадии разработки; оценка извлекаемых запасов; прогноз добычи; фильтр Калмана.

Neft yataqlarının işlənməsinin həyat tsiklinin statistik modelləşdirilməsi

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Xülasə

Statistik təhlil əsasında neft yatağının mərhələlər üzrə strukturlaşdırılması və işlənmə mərhələlərinin sərhədlərinin müəyyənləşdirilməsi metodikası təklif olunmuşdur. Cari neft hasilatının maksimal səviyyəsinə nail olunması vaxtının və çıxarılabilən qalıq ehtiyatların hesablanması prosedurları təqdim olunmuşdur. İşlənmənin son mərhələsində toplam neft hasilatı göstəricilərinə əsasən adaptiv Kalman süzgəcinin köməyi ilə diskret vaxtda neft hasilatının proqnozları qurulmuş və mövcud işlənmə sistemində layihə göstəricilərinə nail olmanın mümkünlüyünün qiymətləri verilmişdir.

Açar sözlər: mərhələlər üzrə strukturlaşdırma; işlənmə mərhələsinin sərhədlərinin ayrılması; çıxarılabilən ehtiyatların qiymətləndirilməsi; hasilat proqnozu; Kalman süzgəci.